Wirth, *Measurability of Definable Sets over Tame Ordered Fields*

In recent years, the interplay between Model Theory and Statistical Learning Theory has increasingly received attention. Notably, Laskowski [1] established a fundamental connection between NIP and the Vapnik–Chervonenkis (VC) dimension, while the Fundamental Theorem of Statistical Learning links VC dimension to probably approximately correct (PAC) learning. When analyzing the learnability of hypothesis spaces definable over tame ordered fields – such as o-minimal expansions of the reals – measurability requirements need to be accounted for.

In this talk, we explore the measurability of definable sets and functions in such settings with a view toward model-theoretic applications of the Fundamental Theorem of Statistical Learning. The structures of particular interest expand linearly ordered sets, as these are naturally endowed with the order-topology. Thus, Borel σ–algebras formed by the Borel sets arising from the order-topology are obvious candidates for our measure-theoretic examination. A central focus will be on identifying sufficient conditions under which definable sets and relevant functions are Borel measurable.

Time permitting, we will touch upon ongoing work concerning measurability subtleties in structures beyond the tame setting.

I report on [2], which is submitted for publication and is part of my doctoral research project supervised by Professor Salma Kuhlmann and Dr. Lothar Sebastian Krapp at University of Konstanz.

References

[1] M. C. Laskowski. Vapnik–Chervonenkis Classes of Definable Sets, *J. Lond. Math. Soc., II. Ser.* 45, 377–384, 1992. `https://doi.org/10.1112/jlms/s2-45.2.377`

[2] L. S. Krapp and L. Wirth. Measurability in the Fundamental Theorem of Statistical Learning. Preprint, 2024, arXiv:2410.10243.