Tressl, *Updates on decidability of real closed rings*

A real closed ring is – roughly speaking – an abstract version of a ring of continuous real valued functions on a topological space. Prominent examples are real closed fields (like the real numbers or the Puiseux series field over the reals), convex valuation rings of real closed fields (like Puiseux series with non-negative support) and rings of continuous functions.

Real closed rings occur in the topological study of semi-algebraic sets [i.e. sets described by polynomial inequalities], where they take on the role of coordinate rings, just like algebras over a field $\mathbb{K}$ are coordinate rings of (affine) varieties defined over $\mathbb{K}$. In this context real closed rings were introduced by Niels Schwartz in the early 1980s.

Decidability of a ring R asks whether there is an algorithm that decides truth in R of any given sentence of the first-order language of rings. Tarski's landmark result says that the real field is decidable and decidability of a real closed ring R addresses in a certain sense the question on how Tarski's theorem can be globalized (think of R as ring of global sections of its Zariski sheaf).

In this talk I will give an overview of known results and report on recent progress.