

**Kuhlmann**, *Arithmetic of cuts in ordered abelian groups and of ideals over valuation rings*

Dedekind cuts are in one-to-one correspondence with their upper cut sets, which are final segments. We investigate existence, uniqueness and maximality of solutions  $T$  for equations  $S_1 + T = S_2$  and inequalities  $S_1 + T \subseteq S_2$  where  $S_1$  and  $S_2$  are final segments of ordered abelian groups. This yields information about the corresponding equalities and inequalities for cuts. We apply our results to investigate existence, uniqueness and maximality of solutions  $J$  for equations  $I_1 J = I_2$  and inequalities  $I_1 J \subseteq I_2$  where  $I_1$  and  $I_2$  are ideals of valuation rings. This enables us to compute the annihilators of quotients of the form  $I_1/I_2$ . The results are applied in [1, 2] to compute the annihilators of Kähler differentials for Galois extensions of prime degree of valued fields.

Partial results in this direction had been obtained by Paulo Ribenboim in [4]. In my talk I will discuss in detail the progress that has been made since his work.

This is joint work with Katarzyna Kuhlmann ([3]).

## References

- [1] S. D. Cutkosky, F.-V. Kuhlmann, A. Rzepka. On the computation of Kähler differentials and characterizations of Galois extensions with independent defect. *Mathematische Nachrichten*, DOI: <https://onlinelibrary.wiley.com/doi/abs/10.1002/mana.202300532>; <https://arxiv.org/abs/2305.10022>
- [2] S. D. Cutkosky, F.-V. Kuhlmann. Kähler differentials of extensions of valuation rings and deeply ramified fields. Submitted. <https://arxiv.org/abs/2306.04967>
- [3] F.-V. Kuhlmann, K. Kuhlmann. Arithmetic of cuts in ordered abelian groups and of ideals over valuation rings. Submitted. <https://arxiv.org/abs/2406.10545>
- [4] P. Ribenboim. Sur les groupes totalement ordonnés et l'arithmétique des anneaux de valuation. *Summa Brasil. Math.* 4, 1–64, 1958.