Krapp, *Definable Ranks*

Originally studied in the context of valued fields (see [1, Section 2.1]), ranks function as order-theoretic invariants for linearly ordered structures of varying algebraic complexity. Most generally, the rank of an ordered algebraic structure is the order type of a certain collection of convex substructures, ordered by set inclusion: For an ordered field, this collection consists of all of proper convex subrings (i.e. the non-trivial convex valuation rings of that field); for ordered abelian groups, it consists of all proper convex subgroups; for ordered sets (without any further algebraic structure), the collection contains all proper final segments. Studying the ranks of ordered fields, ordered abelian groups and ordered sets becomes of special interest in the presence of valuations. More specifically, the finest convex valuation on an ordered field — its so-called natural valuation — connects a unique ordered abelian group to that field. Likewise, each ordered abelian group has a natural valuation resulting in an ordered value set. Now the rank of any ordered field coincides with the rank of its value group under the natural valuation; likewise the rank of any ordered abelian group is identical to the rank of its value set under the natural valuation. Thus, for an ordered field, all three ranks coincide: at the field, the value group and the value set levels. From a model-theoretic perspective, ranks can be limited to collections of convex substructures that are first-order definable, introducing the concept of definable ranks. For instance, the definable rank of an ordered field is the order type of the set of its first-order definable convex valuation rings. While it is readily established that definable ranks generally do not coincide at the field, value group, and value set levels, the question remains as to what order-theoretic relationships still exist between them. In my talk, I will begin by outlining some considerations regarding definable ranks at the field level, connecting this to the study of definable valuation rings and the works of Prestel, e.g. [3,4]. I will then present some preliminary results from joint work with Salma Kuhlmann and Lasse Vogel [2], which specifically examines the definable ranks of almost real closed fields at field, group and set levels.

References

[1] A. J. Engler and A. Prestel. *Valued Fields.* Springer (Berlin), 2005.

[2] L. S. Krapp, S. Kuhlmann and L. Vogel. Definable Ranks in Almost Real Closed Fields. In preparation.

[3] A. Prestel and M. Ziegler. Model-theoretic methods in the theory of topological fields. *J. Reine Angew. Math.* 299/300, 318—341, 1978.

[4] A. Prestel. Definable Henselian Valuation Rings. *J. Symb. Log.* 80, 1260—1267, 2015.