Hubert, *Orbit separation and stratification by isotropy classes of piezoelectricity tensors*

Invariants are essential for classifying mathematical objects up to a group of transformations. For a compact Lie group, there is always a finite set of polynomial invariants that separate all the orbits. Yet such a set is challenging to compute and can have high cardinality. We consider here the representation of the group $\mathrm{SO}_3(\mathbb{R})$ on the space $\mathbb{P}\mathrm{iez}$ of piezoelectricity tensors to illustrate a different approach to separate orbits and classify their isotropy.

We shall stratify the space $\mathbb{P}\mathrm{iez}$ into $\mathrm{SO}_3(\mathbb{R})$-invariant semi-algebraic sets that are each the orbit of a strong Seshadri slice. A separating set of invariants for the action of the normalizer on the slice can be lifted to a separating set of invariants for the action of $\mathrm{SO}_3(\mathbb{R})$ on the semi-algebraic set. The isotropy classes of the orbits can also be decided on the slice.

The space of piezoelectricity tensors decomposes into $\mathrm{SO}_3(\mathbb{R})$-irreducible representations as $\mathbb{P}\mathrm{iez} \cong \mathcal{H}_1 \oplus \mathcal{H}_1 \oplus \mathcal{H}_2 \oplus \mathcal{H}_3$, where $\mathcal{H}_d$ is the vector space of homogeneous harmonic polynomials of degree d. We present and make use of strong Seshadri slices on $\mathcal{H}_1$, $\mathcal{H}_2$ and the subvariety of $\mathcal{H}_2$ that corresponds to tracefree symmetric matrices with a double eigenvalue. The normalizers are either $\mathrm{O}_2(\mathbb{R})$ or the octahedral group (a.k.a. the group of signed permutations). Separating sets of invariants can be systematically determined for the actions of these groups and have a moderate cardinality. Our approach could thus be extended to other representations of $\mathrm{SO}_3(\mathbb{R})$, as for instance the space of elasticity tensors.

References

E. Hubert and M. Jalard. Orbit separation and stratification by isotropy classes of piezoelectricity tensors. `https://hal.science/hal-04905290`