Guier, *Elimination of quantifiers in the theory of projectable real closed rings with first convexity property*

The main purpose of this talk will be to give the characterization of projectable reduced f-rings that are divisible-projectable satisfying the first convexity property and admitting elimination of quantifiers in the language of lattice-ordered rings with the divisibility relation, the radical relation associated to the minimal prime spectrum and the local divisibility relation. This latter relation was introduced in [3], in order to prove that the class of previous rings that are real closed, sc-regular and without non-zero minimal idempotents admits elimination of quantifiers in this new language. I will first discuss this result. Following the same technics, it is easily proved in [4] that two other theories of real closed rings also admit q.e. in this language. The elimination of quantifiers of von Neumann regular real closed rings without non-zero minimal idempotents developed in [6] is at the origins of the results in [3], and should also be consider in this context (the divisibility is the radical relation and the local divisibility turns to be trivial). The mentioned characterization can be achieved using a result in [2] (based on earlier results of F. Point, cf. [5]).

References

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