D'Elbée, *Lie methods for ω-categorical Engel groups*

In 1981, Wilson conjectured that any omega-categorical locally nilpotent p-group is nilpotent. If true, a quite satisfactory decomposition of every ω-categorical groups would follow. This conjecture is very much open more than 40 years later.

The analogue statement for Lie algebras (every locally nilpotent ω-categorical Lie algebra is nilpotent) is also open. Both statement can be reformulated as: is any ω-categorical Engel group/Lie algebra nilpotent. As such, those questions are connected to Burnside-type problems and the work of Higman, Kostrikin, Zelmanov, Vaughan-Lee, Traustason, etc. For instance, using a classical result of Zelmanov, the conjecture for Lie algebras is true asymptotically in the following sense: for each n, every n-Engel Lie algebra over $\mathbb{F}_p$ is nilpotent for all but finitely many p's. There is a similar statement for groups. The situation for small values of the pair (n, p) is highly characteristic-dependent, for instance, 4-Engel Lie algebras over a field of characteristic p are nilpotent except if $p = 2, 3$ or $p = 5$. I recently proved that ω-categorical n-Engel Lie algebras over a field of characteristic p are nilpotent for $(n, p) = (3, 5)$ and $(n, p) = (4, 3)$. In this talk I will explain how to use those results to deduce that ω-categorical 3-Engel p-groups and 4-Engel p-groups (with p odd) are nilpotent. This work is at the intersection of model theory, group theory and computer algebra.