Degtyarev, *Split hyperplane sections on polarized K3-surfaces*

I will discuss a new result which is an unexpected outcome, following a question by Igor Dolgachev, of a long saga about smooth rational curves on (quasi-)polarized K3-surfaces. The best known example of a K3-surface is a quartic surface in space. A simple dimension count shows that a typical quartic contains no lines. Obviously, some of them do and, according to B. Segre, the maximal number is 64 (an example is to be worked out). The key rôle in Segre's proof (as well as those by other authors) is played by plane sections that split completely into four lines, constituting the dual adjacency graph K(4). A similar, though less used, phenomenon happens for sextic K3-surfaces in $\mathbb{P}^4$ (complete intersections of a quadric and a cubic): a split hyperplane section consists of six lines, three from each of the two rulings, on a hyperboloid (the section of the quadric), thus constituting a K(3, 3).

Going further, in degrees 8 and 10 one's sense of beauty suggests that the graphs should be the 1-skeleton of a 3-cube and Petersen graph, respectfully. However, further advances to higher degrees required a systematic study of such 3-regular graphs and, to my great surprise, I discovered that typically there is more than one! Even for sextics one can also imagine the 3-prism (occurring when the hyperboloid itself splits into two planes).

The ultimate outcome of this work is the complete classification of the graphs that occur as split hyperplane sections (a few dozens) and that of the configurations of split sections within a single surface (manageable starting from degree 10). In particular, answering Igor's original question, the maximal number of split sections 4 on a quartic is 72, whereas on a sextic in $\mathbb{P}^4$ it is 40 or 76, depending on the question asked. The ultimate champion is the Kummer surface of degree 12: it has 90 split hyperplane sections.

The tools used (probably, not to be mentioned) are a fusion of graph theory and number theory, sewn together by the geometric insight.