De Mase, *Augmentable ordered abelian groups and definable henselian valuations*

By the results of Schmitt [3] and Cluckers-Halupczok [2], many first-order properties of ordered abelian groups can be reduced to corresponding properties of their *spines*, which are chains of uniformly definable convex subgroups. We investigate the property of augmentability by infinite elements for ordered abelian groups. Specifically, we say that an ordered abelian group G is *augmentable by infinite elements* if there exists an ordered abelian group H such that $G \preceq H \oplus G$. Via reduction to the spines, we show that every non-trivial ordered abelian group is augmentable by infinite elements. This result has implications for the study of definable henselian valuations. In particular, we show that a field k of characteristic zero is not t-henselian (i.e., not elementarily equivalent to any field admitting a non-trivial henselian valuation) if and only if all henselian valuations with residue field k are ($\emptyset$-)definable in the language of rings.

This is joint work with B. Boissonneau, F. Jahnke, and P. Touchard [1].

References

[1] B. Boissonneau, A. De Mase, F. Jahnke, P. Touchard. Growing spines ad infinitum. arXiv:2501.10531 [math.LO], 2025.

[2] R. Cluckers and I. Halupczok. Quantifier elimination in ordered abelian groups. *Confluentes Math.*, 3(4):587-–615, 2011.

[3] P. H. Schmitt. *Model theory of ordered abelian groups*, 1982. Habilitationsschrift.