Berarducci, *Projective curves and weak second-order logic*

This is joint work with Francesco Gallinaro.

Given an algebraically closed field $\mathbb{K}$ of characteristic zero, we study the incidence relation between points and irreducible projective curves, or more precisely the poset $\mathrm{Pos}(\mathbb{K})$ of irreducible proper sub-varieties of the projective plane over $\mathbb{K}$. Answering a question of Marcus Tressl, we prove that the poset interprets the field $\mathbb{K}$ and the subring of integers, and it is in fact bi-interpretable with the two-sorted structure $(\mathbb{K}, \mathrm{Fin}(\mathbb{K}))$ consisting of the field $\mathbb{K}$ and a sort for its finite subsets. We prove that the theory of the poset depends on the transcendence degree of $\mathbb{K}$, so for instance the complex numbers and the algebraic numbers have non-elementary-equivalent posets. Taking for $\mathbb{K}$ the complex numbers, we give a complete recursive axiomatization of $(\mathbb{K}, \mathrm{Fin}(\mathbb{K}))$ modulo the theory of the integers. We also show that the integers are stably embedded in this structure.